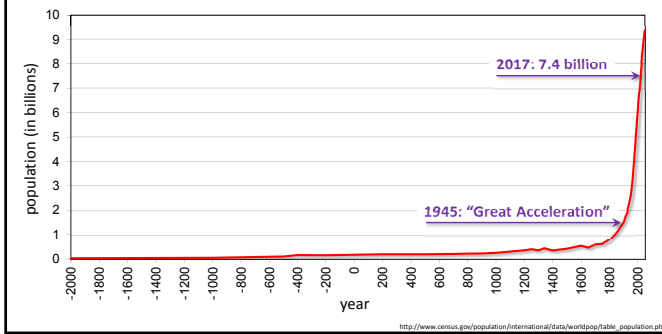
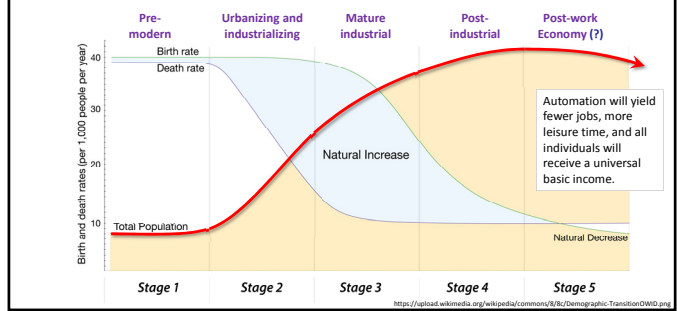


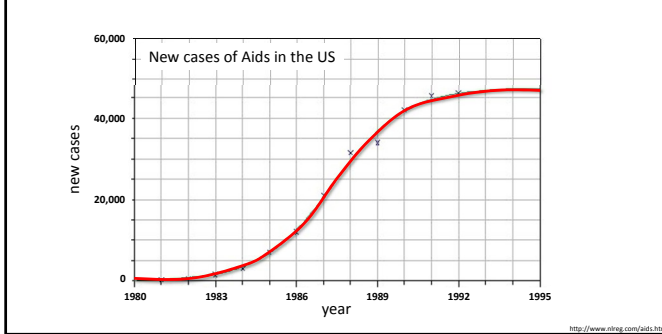
**Rapid growth in the number of people on Earth is unabated.**



**The Demographic Transition Model represents one way to describe the evolution of societies.**



**Sigmoidal curves, S-curves, have an inversion point.**



**A reminder about percent, fractions, and decimals...**

10% = 0.1

1% = 0.01

1/10% = 0.1% = 0.001

1/10% = 0.1% = 0.001 = 1‰

Note: always put a zero (or a number) in front of a decimal point, and always use a space before the units (except for most symbols).

**A reminder about the distributive law...**

1. Yank out what is common between the pluses (+) and minuses (-).
2. Put it in front of the parentheses.
3. Put what is left inside the parentheses.

**Discrete Compounding**

$$N_t = N_0(1+\lambda)^t$$

$N_0$  = initial amount  
 $\lambda$  = rate of natural increase  
 $t$  = time interval

### Discrete Compounding

$$N_t = N_0(1 + \lambda/m)^{mt}$$

- $N_0$  = initial amount
- $\lambda$  = rate of natural increase
- $t$  = time interval
- $m$  = number of compounding events during one time interval

### Continuous Compounding

$$N_t = N_0 e^{\lambda t}$$

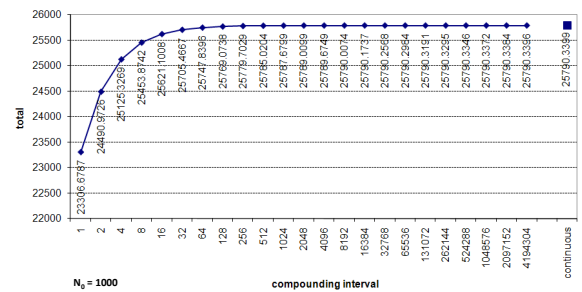
- $N_0$  = initial amount
- $e$  = the natural log
- $\lambda$  = rate of natural increase
- $t$  = time interval

Calculus introduces the concepts of limits...

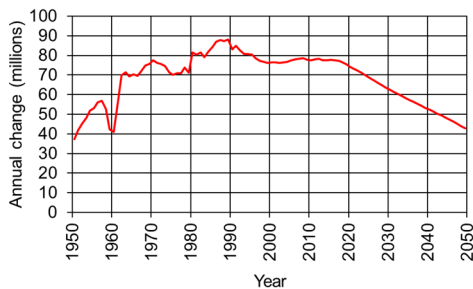
As the number of compounding events increases, the compounding interval becomes infinitesimally small, and the function becomes continuous; this is calculus.

[spreadsheet: limits](#)

An example of the relationship between compounding interval and total value after 50 years of growth at 6.5%.



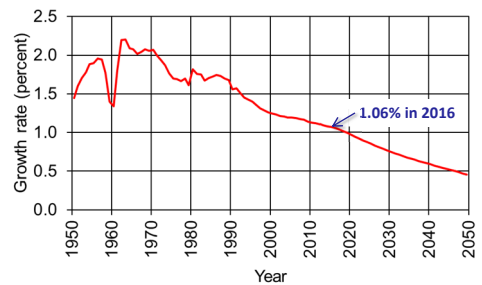
Annual world population change: 1950 to 2050.



Aug 2016

<https://www.census.gov/population/international/data/db/worldpopgraph.php>

Annual world population growth rates: 1950 to 2050.



Aug 2016

<https://www.census.gov/population/international/data/db/worldpopgraph.php>